

## Semester 2 (Unit 1 & Unit 2) Examination, 2017

### Question/Answer Booklet

## MATHEMATICS SPECIALIST

### Section One: Calculator-free

Student Name/Number: \_\_\_\_\_

Teacher Name: \_\_\_\_\_

#### Time allowed for this section

Reading time before commencing work: five minutes

Working time for this section: fifty minutes

#### Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet  
Formula Sheet

#### To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

#### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

### Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	9	9	50	50	35
Section Two: Calculator-assumed	15	13	100	100	65
					100

### Instructions to candidates

- The rules for the conduct of School exams are detailed in the \_\_\_\_\_ *School/College assessment policy*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

## Section One: Calculator-free

35% (50 Marks)

This section has **9** questions. Answer **all** questions. Write your answers in the spaces provided. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Suggested working time: **50 minutes**.

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**Question 1****(6 marks)**

If  $\underline{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$  and  $\underline{b} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ , determine

(a)  $\underline{a} - 3\underline{b}$ . (1 mark)

(b)  $|\underline{a}|$ . (1 mark)

(c) a unit vector in the same direction as  $\underline{a}$ . (2 marks)

(d) a vector with the same magnitude as  $\underline{a}$  but in the same direction as  $\underline{b}$ . (2 marks)

**Question 2****(4 marks)**

Points A, B and C have position vectors  $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ ,  $\begin{pmatrix} -1 \\ 15 \end{pmatrix}$  and  $\begin{pmatrix} 9 \\ -25 \end{pmatrix}$  respectively. Use vectors to prove that A, B and C are collinear.

**Question 3****(4 marks)**

(a) Determine the exact complex roots to the equation  $2x^2 + 6x + 5 = 0$ . (2 marks)

(b) Simplify  $\frac{3+i}{7i-1}$  (2 marks)

**Question 4****(4 marks)**

Given that  $z = a + bi$ ,  $w = c + di$ ,  $\bar{z}$  is the conjugate of  $z$  and  $\bar{w}$  is the conjugate of  $w$ , prove that the following is true.

$$\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$$

## Question 5

(9 marks)

- (a) How many different ways can 6 people, consisting of 1 woman, 2 men, a couple and their teenage daughter, form a queue if the couple and their daughter are to be together? (2 marks)
- (b) If  ${}^{20}C_{r-8} = {}^{20}C_r$ , evaluate the value of  $r$ . (2 marks)
- (c) Find the integer  $m$  if  ${}^mP_{m-4} = 210$ . (1 mark)
- (d) Find the number of 4-letter code words that can be made from the letters in the word CHOCOLATES. (4 marks)

**Question 6****(5 marks)**(a) Prove that  $\cot x - \cot 2x = \operatorname{cosec} 2x$ **(3 marks)**(b) Express  $\sqrt{2}(\cos x + \sin x)$  in the form  $R\cos(x - \alpha)$  such that  $R > 0$  and  $\alpha$  is an acute angle.**(2 marks)**

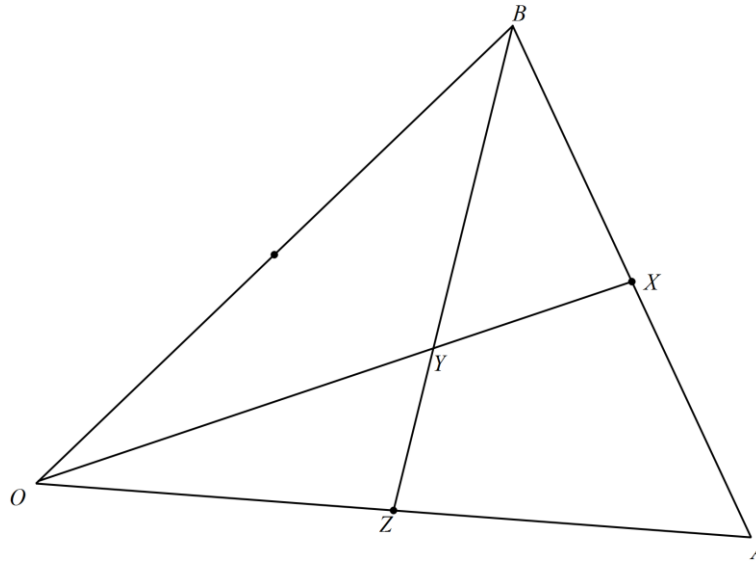
Question 7

(5 marks)

In the triangle OAB two medians are drawn with X and Z being midpoints of their sides. These two medians intersect at point Y.

Let  $\vec{OA} = \underline{a}$  and  $\vec{OB} = \underline{b}$  and  $\vec{OY} = h\vec{OX}$  and  $\vec{BY} = k\vec{BZ}$ .

By using the vector equation  $\vec{OB} = \vec{OY} + \vec{YB}$  prove that  $h = k = \frac{2}{3}$  (i.e prove that the medians of a triangle intersect at a point two-thirds the way along)





## Question 8

(6 marks)

$$\text{Given } A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

(a) Evaluate  $A^{-1}$ .

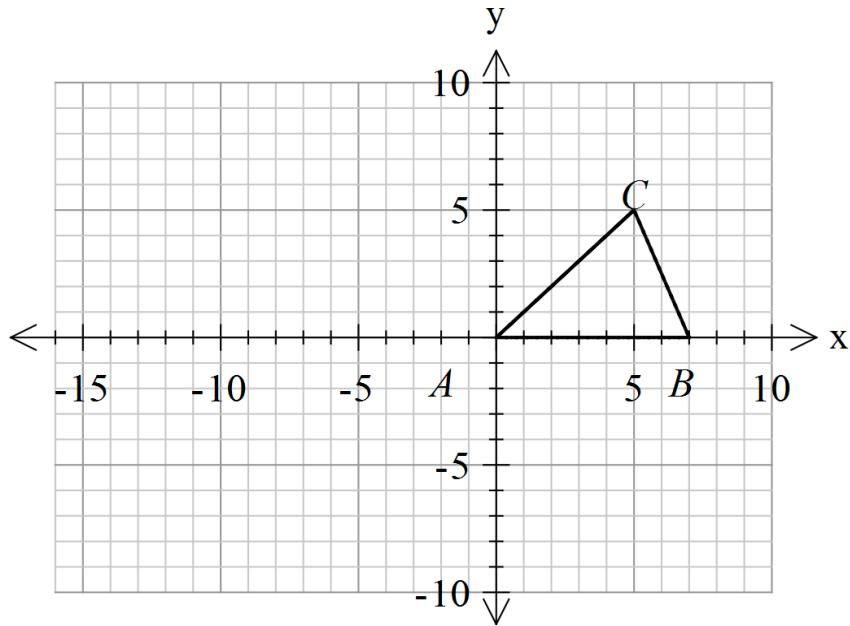
(2 marks)

(b) Solve the system  $\begin{cases} x + 3y = 13 \\ 4x + 2y = 2 \end{cases}$  clearly showing your use of  $A^{-1}$ .

(4 marks)

Question 9

(7 marks)



The triangle ABC, see above, is transformed by a reflection in the line  $y = -x$  and is then followed by a shear of factor 2 parallel to the  $x$  axis.

(a) Represent the double transformation by a single matrix. (3 marks)

(b) Determine the exact area of the final shape. (2 marks)

(c) Determine the matrix that will return the final image in part(a) back to the original triangle. (2 marks)

End of Questions

Additional working space

Question number: \_\_\_\_\_

**Acknowledgements**

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